From a finite element model to system level simulation by means of model reduction

MUMPS Users Meeting 2010, Toulouse

Evgenii Rudnyi, CADFEM GmbH

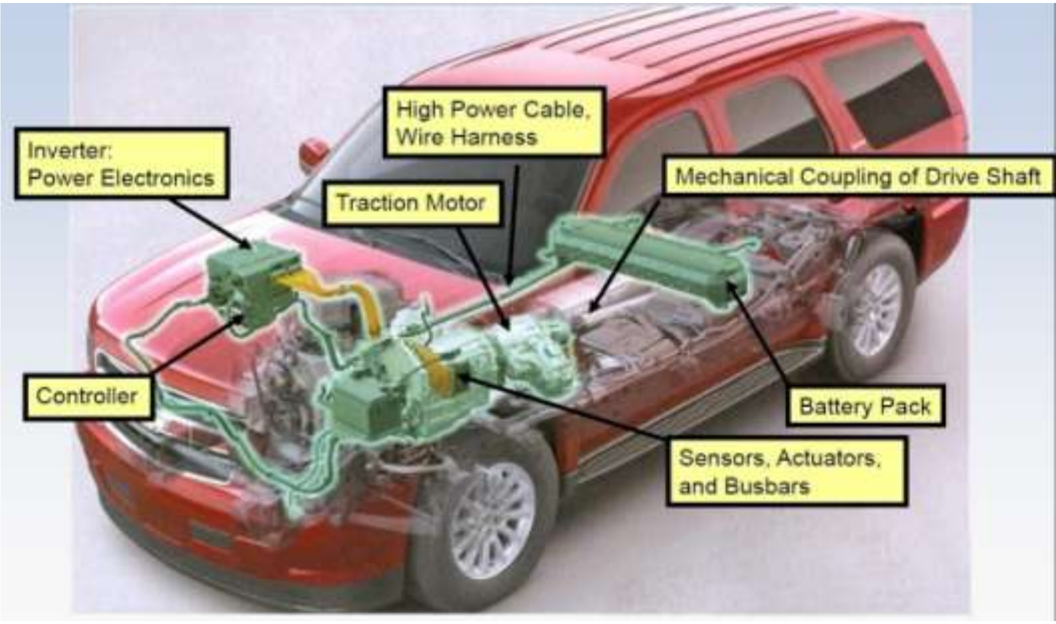
erudnyi@cadfem.de

http://ModelReduction.com

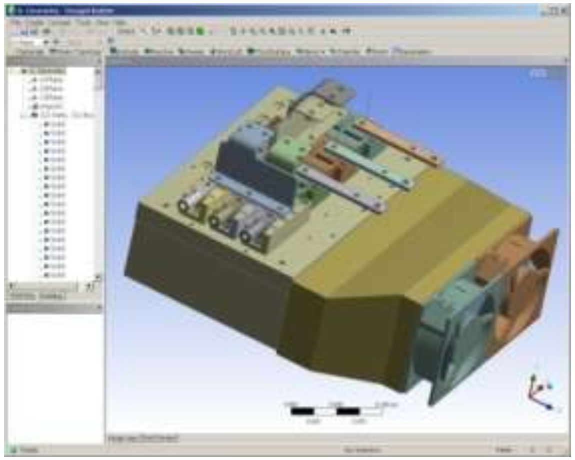
CADFEM

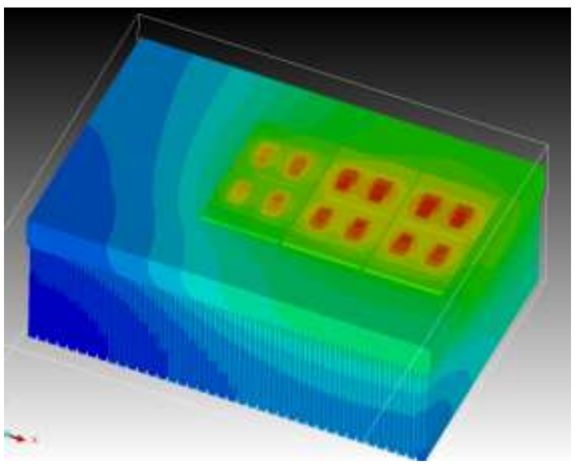
Content

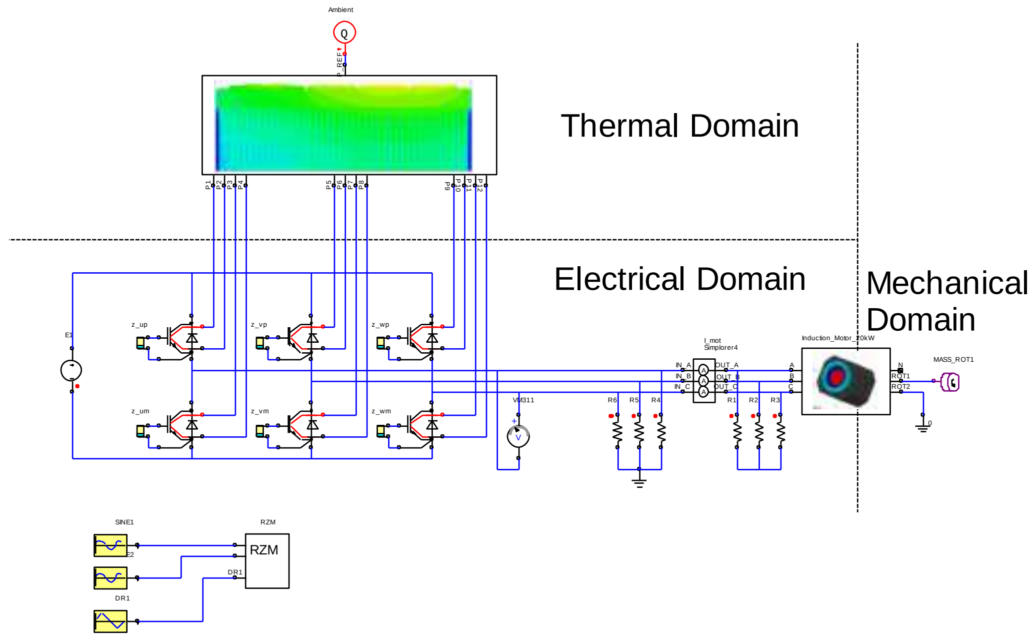
- System level simulation and compact modeling
- Model order reduction
- Case studies with MOR for ANSYS
- Experience with MUMPS solver



Device and System Level Simulation







- How to make a model at system level from that at device level?

Compact Modeling: Transistor Compact Model

$$I_E = I_{F0}(e^{qV_{EB}/kT} - 1) - \alpha_R I_{R0}(e^{qV_{CB}/kT} - 1)$$
$$I_E = \alpha_F I_{F0}(e^{qV_{EB}/kT} - 1) - I_{R0}(e^{qV_{CB}/kT} - 1)$$

Too much reliance on intuition

$$-\varepsilon \nabla^2 \Psi = q(p - n + N_0)$$
$$\frac{\partial n}{\partial t} = \nabla \bullet (-\mu_n n \nabla \Psi + D_n \nabla n) - R_n$$
$$\frac{\partial p}{\partial t} = \nabla \bullet (\ \mu_p p \nabla \Psi + D_p \nabla p) - R_p$$

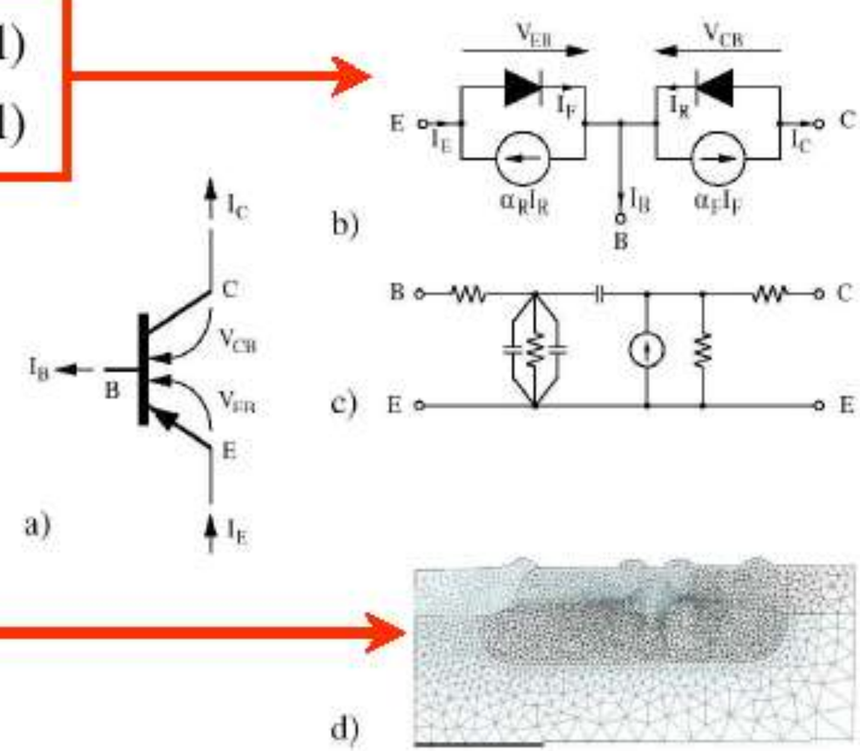
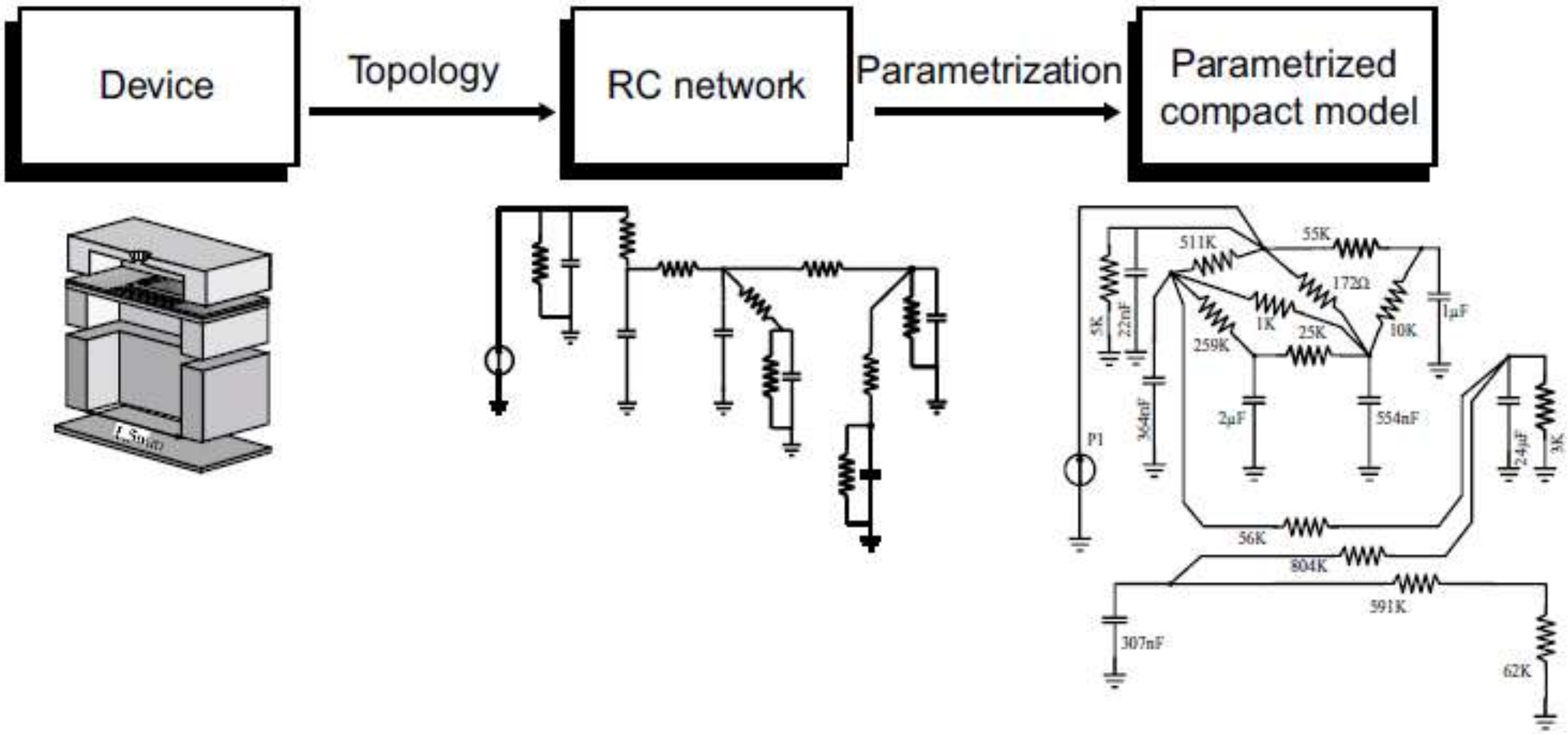


Figure from J. Lienemann, E. B. Rudnyi and J. G. Korvink. MST MEMS model order reduction: Requirements and Benchmarks. Linear Algebra and its Applications, v. 415, N 2-3, p. 469-498, 2006.

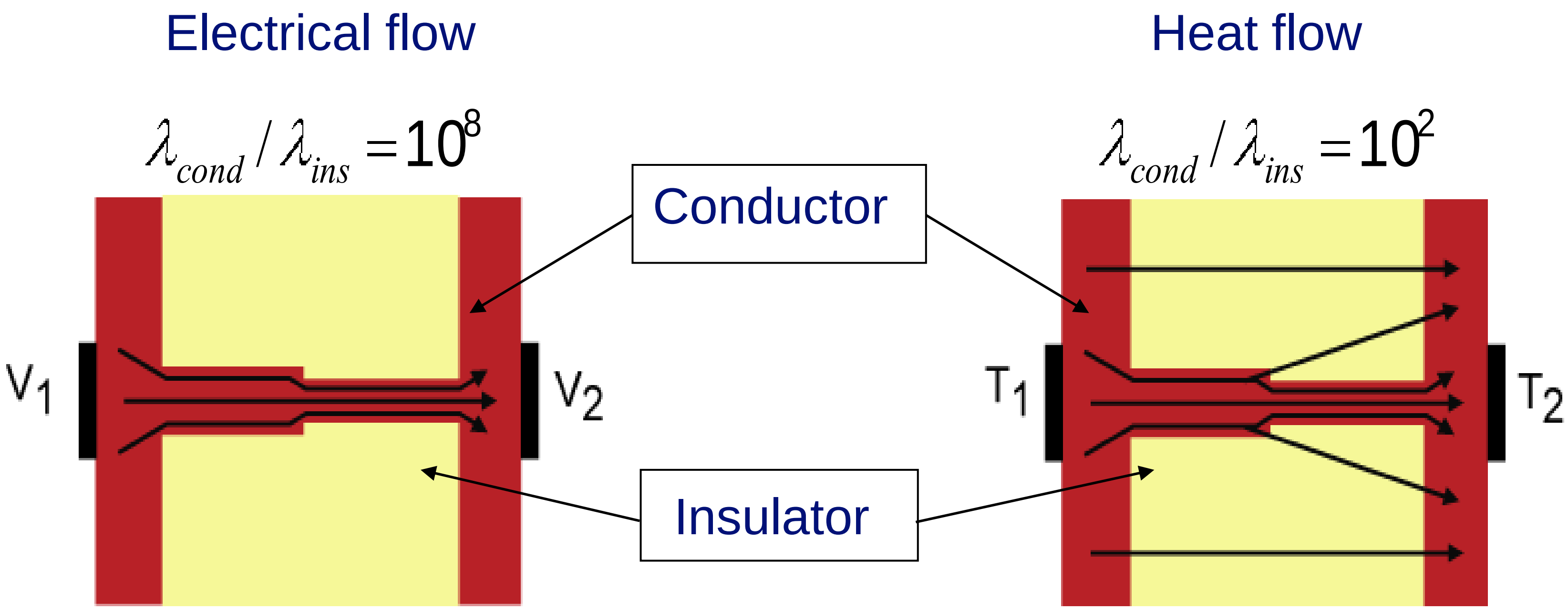
Compact Thermal Models



- Looks understandable – but how to do it in practice?

Figure from the book "*Fast Simulation of Electro-Thermal MEMS: Efficient Dynamic Compact Models.*" *Springer, 2006.*

Comparison: Electrical vs. Thermal

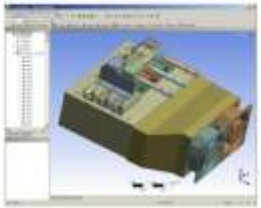


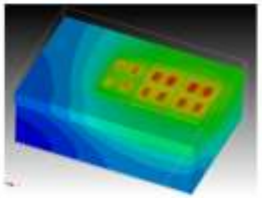
- Thermal phenomena are much more distributed, it is hard to lump them.

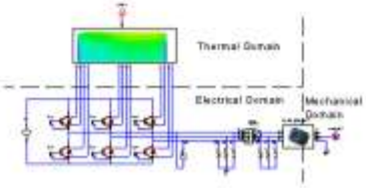
Figure from the book "*Fast Simulation of Electro-Thermal MEMS: Efficient Dynamic Compact Models.*" *Springer, 2006.*

Content

- System level simulation and compact modeling
- Model order reduction
- Case studies with MOR for ANSYS
- Experience with MUMPS solver



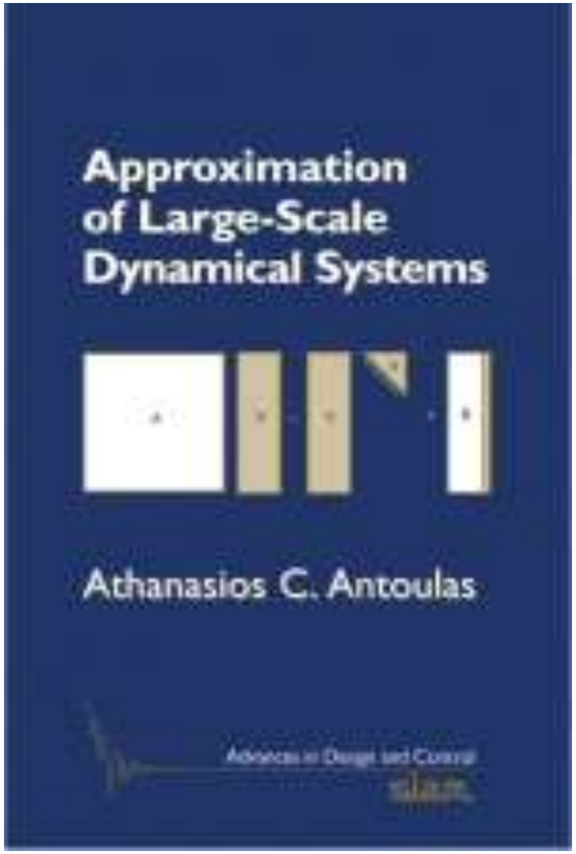




CADFEM

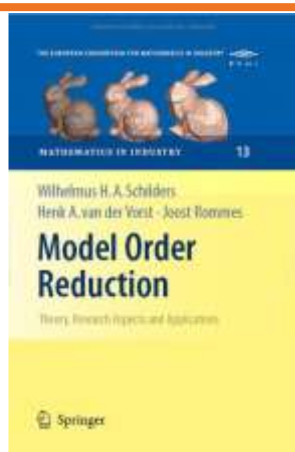
Model Order Reduction

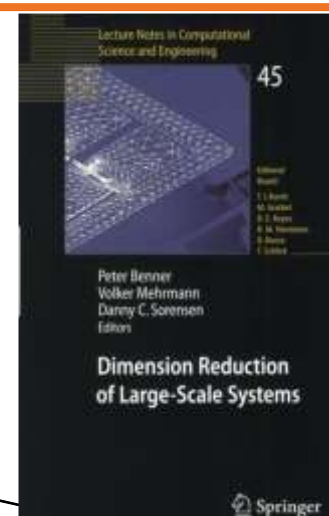
- Relatively new technology:
  - It is not mode superposition;
  - It is not Guyan reduction;
  - It is not CMS.
- Solid mathematical background:
  - Approximation of large scale dynamic systems
- Dynamic simulation:
  - Harmonic or transient simulation
- Industry application level:
  - Linear dynamic systems



CADFEM

Linear Model Order Reduction





Linear Dynamic System, ODEs

Control Theory BTA, HNA, SPA $O(N^3)$

Moment Matching via Krylov subspaces. Iterative, based on matrix vector product.

Low-rank Grammian, SVD-Krylov

one-side Arnoldi process

double-side Lancsoz algorithm

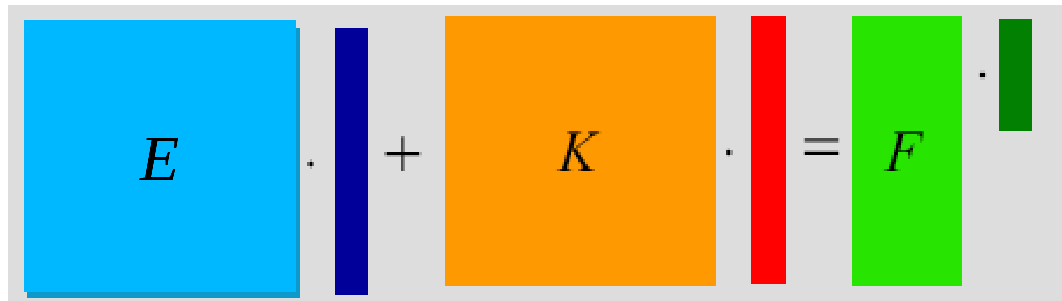
MOR for ANSYS

CADFEM

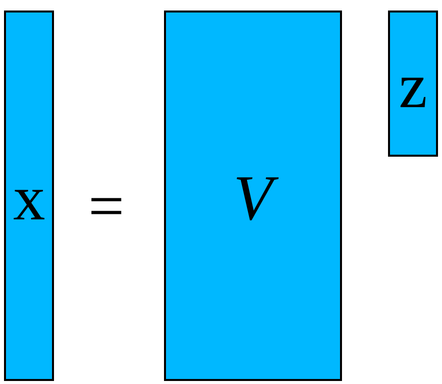
Model Reduction as Projection

- Projection onto low-dimensional subspace

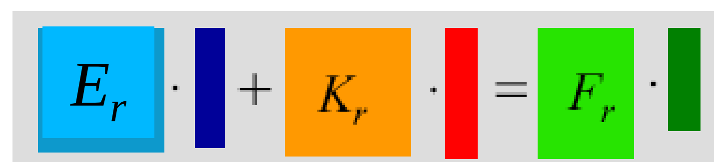
$$E\dot{\mathbf{x}} + K\mathbf{x} = B\mathbf{u}$$



$$\mathbf{x} = V\mathbf{z} + \varepsilon$$



$$V^T E V \dot{\mathbf{z}} + V^T K V \mathbf{z} = V^T B \mathbf{u}$$



- How to find subspace?
- Mode superposition is not the best way to do it.

Hankel Singular Values

- Dynamic system in the state-space form:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$H(s) = C(sI - A)^{-1} B$$

- Lyapunov equations to determine controllability and observability Grammians:

$$AP + PA^T + BB^T = 0$$

$$A^T Q + QA + C^T C = 0$$

- Hankel singular values (HSV):
  - square root from eingenvalues for product of Grammians.

$$\sigma_i = \sqrt{\lambda_i(PQ)}$$

CADFEM

Global Error Estimate

- Infinity norm

$$\left\| H(s) - \hat{H}(s) \right\|_\infty =$$

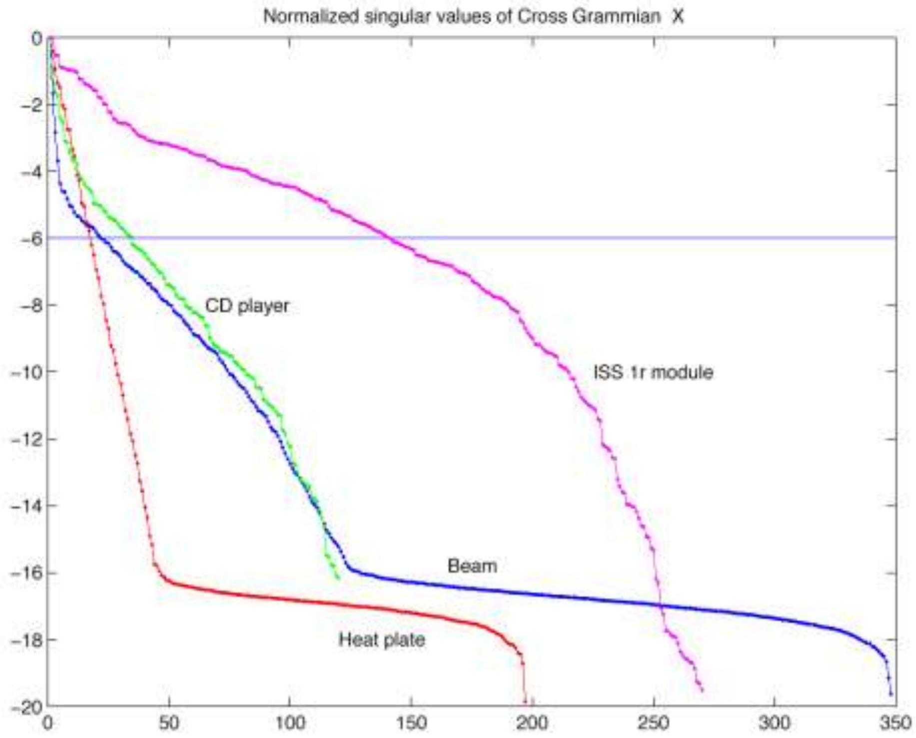
$$= \max_s abs(H(s) - \hat{H}(s))$$

- Global error for a reduced model of dimension k

$$\left\| H(s) - \hat{H}(s) \right\|_\infty <$$

$$2(\sigma_{k+1} + \ldots + \sigma_n)$$

- Model reduction success depends on the decay of HSV.



- Log10[HSV(i)] vs. its number.
- From Antoulas review.

CADFEM

Implicit Moment Matching

- Padé approximation
- Matching first moments for the transfer function

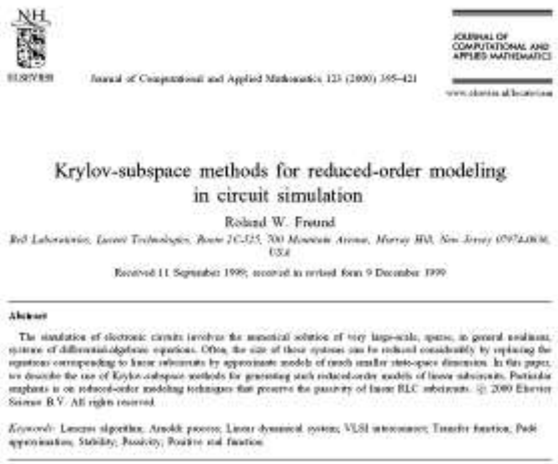
$$E\dot{\mathbf{x}} + K\mathbf{x} = B\mathbf{u}$$

$$H(s) = (sE + K)^{-1} B$$

$$H = \sum_{0}^{\infty} m_i (s - s_0)^i$$

$$H_{red} = \sum_{0}^{\infty} m_{i,red} (s - s_0)^i$$

$$m_i = m_{i,red}, \quad i = 0, \ldots, r$$



ELSEVIER Journal of Computational and Applied Mathematics 123 (2000) 395–421

JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

www.elsevier.nl/locate/cam

Krylov-subspace methods for reduced-order modeling in circuit simulation

Roland W. Freund

Bell Laboratories, Lucent Technologies, Room 2C-525, 700 Mountain Avenue, Murray Hill, New Jersey 07974-0636, USA

Received 11 September 1999; received in revised form 9 December 1999

Abstract

The simulation of electronic circuits involves the numerical solution of very large-scale, sparse, in general nonlinear, systems of differential-algebraic equations. Often, the size of these systems can be reduced considerably by replacing the equations corresponding to linear subcircuits by approximate models of much smaller state-space dimension. In this paper, we describe the use of Krylov-subspace methods for generating such reduced-order models of linear subcircuits. Particular emphasis is on reduced-order modeling techniques that preserve the passivity of linear RLC subcircuits.

Keywords: Lanczos algorithm; Arnoldi process; Linear dynamical system; VLSI interconnect; Transfer function; Padé approximation; Stability; Passivity; Positive real function

- Implicit Moment Matching:
  - via Krylov Subspace

$$s_0 = 0$$

$$V = span\{\Im(K^{-1}E, K^{-1}b)\}$$

CADFEM

Second Order Systems

$$M\ddot{x} + E\dot{x} + Kx = Bu$$
$$y = Cx$$

default → Ignore the damping matrix during MOR: Proportional Damping

-1 → Transform to 1st order system.

-2 → Use Second Order ARnoldi.

Sensors and Materials, Vol. 19, No. 3 (2007) 149–164
MYU Tokyo

S & M 0672

Parametric Order Reduction of Proportionally Damped Second-Order Systems

Rudy Eid*, Behnam Salimbahrami, Boris Lohmann, Evgenii B. Rudnyi[1] and Jan G. Korvink[1]

Institute of Automatic Control, Technical University of Munich, Boltzmannstr 15, 85748 Garching, Germany
[1]Institute for Microsystem Technology (IMTEK), University of Freiburg, 79085 Freiburg, Germany

(Received October 25, 2006; accepted February 22, 2007)

Key words: order reduction, second-order systems, proportional damping, moment matching, second-order Krylov subspaces

SIAM J. SCI. COMPUT.
Vol. 26, No. 5, pp. 1692–1709

DIMENSION REDUCTION OF LARGE-SCALE SECOND-ORDER DYNAMICAL SYSTEMS VIA A SECOND-ORDER ARNOLDI METHOD*

ZHAOJUN BAI† AND YANGFENG SU‡

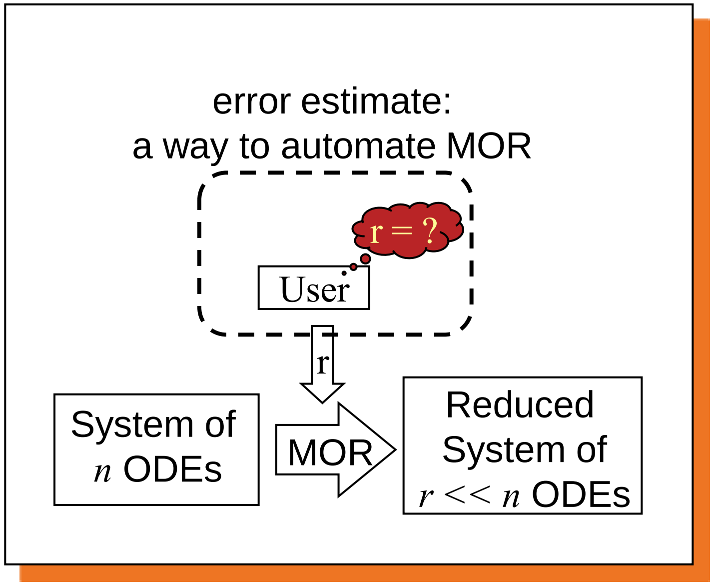
Abstract. A structure-preserving dimension reduction algorithm for large-scale second-order dynamical systems is presented. It is a projection method based on a second-order Krylov subspace. A second-order Arnoldi (SOAR) method is used to generate an orthonormal basis of the projection subspace. The reduced system not only preserves the second-order structure but also has the same order of approximation as the standard Arnoldi-based Krylov subspace method via linearization. The superior numerical properties of the SOAR-based method are demonstrated by examples from structural dynamics and microelectromechanical systems.

Error Indicator

- **Key question :**
  What is a suitable order of the reduced system for a desired accuracy?

- “Rule of thumb”: r = 30-50

- Proposed engineering approach:
  - comparison of reduced systems of order r and $r + 1$



Institute of Physics Publishing
Journal of Micromechanics and Microengineering

J. Micromech. Microeng. 15 (2005) 430–440
doi:10.1088/0960-1317/15/3/002

Error indicators for fully automatic extraction of heat-transfer macromodels for MEMS

Tamara Bechtold, Evgenii B Rudnyi and Jan G Korvink

IMTEK, University of Freiburg, Georges-Köhler-Allee 103, 79110 Freiburg, Germany

E-mail: bechtold@imtek.de

Received 14 June 2004, in final form 15 October 2004
Published 16 December 2004

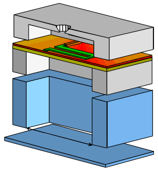
Convergence of Relative Error

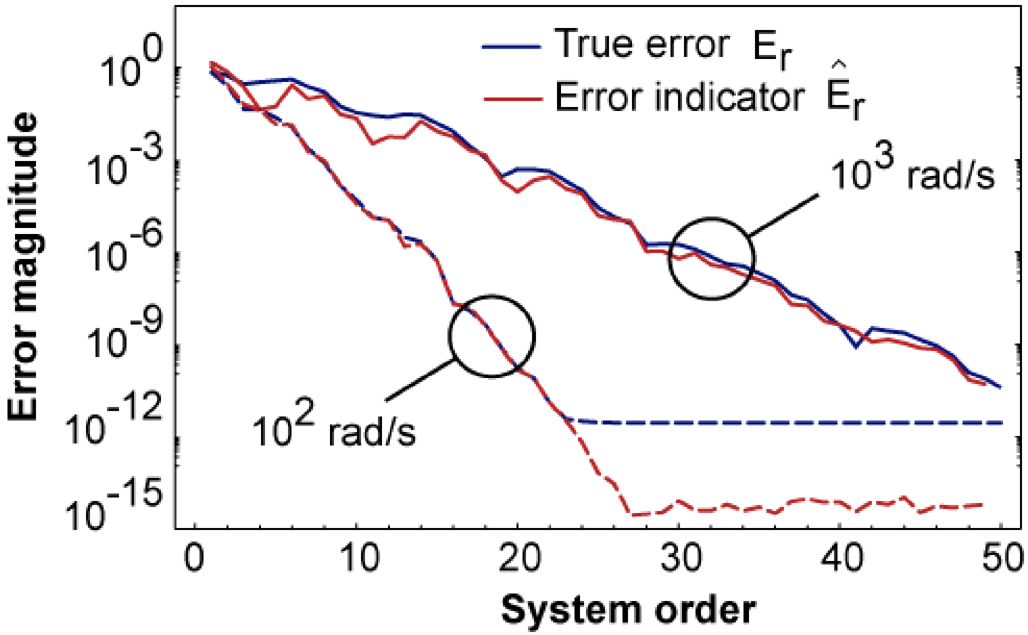
True error: $$E_r(s) = \frac{|H(s) - H_r(s)|}{|H(s)|}$$

Error indicator: $$\hat{E}_r(s) = \frac{|H_r(s) - H_{r+1}(s)|}{|H_r(s)|}$$

Main result:

$$E_r(s) \approx \hat{E}_r(s)$$





CADFEM

Nonlinear Input

$$R = R(T) = R_0 \cdot (1 + \alpha T + \beta T^2 + \ldots)$$

Nonlinear input

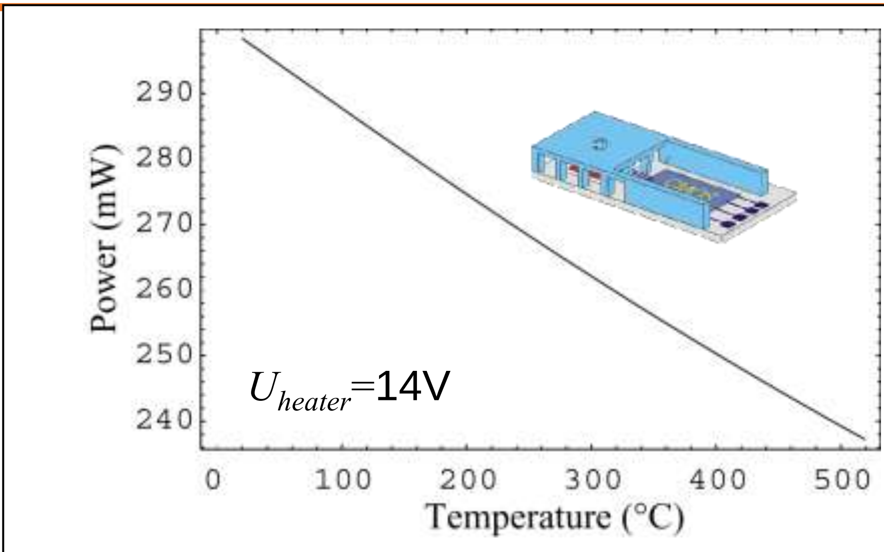
$$\underline{C} \cdot \dot{\underline{T}} + \underline{K} \cdot \underline{T} = F \cdot Q(t,T) = F \cdot \frac{U^2(t)}{R(T)}$$

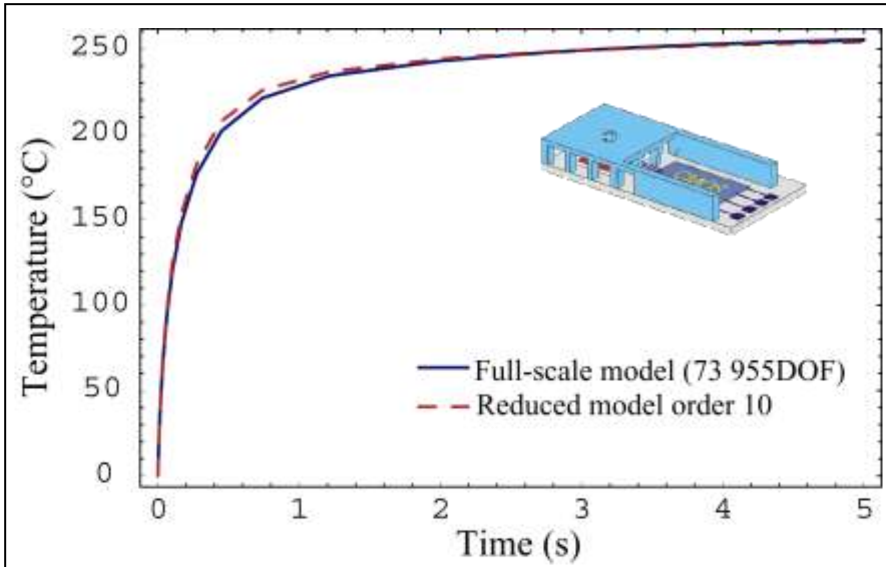
Input function does not participate in MOR

$$V^T C V \cdot \dot{z} + V^T K V \cdot z = V^T F \cdot Q(t, V^* \cdot z)$$

Control temperature is a single node temperature T_N

$$Q(t,T) = Q(T_N(t)) = Q(V^* \cdot z)$$





Information on ModelReduction.com

Model Reduction - Windows Internet Explorer

http://modelreduction.com/ModelReduction/

ModelReduction.com

Discussion group for model reduction: http://groups.google.com/group/mor4ansys. You can subscribe on-line or by sending a dummy e-mail to mor4ansys-subscribe@googlegroups.com.

Book *Fast Simulation of Electro-Thermal MEMS: Efficient Dynamic Compact Models*. Book at Amazon or Springer.

Home	Model Reduction	Applications	Teaching	MOR for ANSYS

MOR Home	Linear	Second Order	Parametric	Weakly Nonlinear	Nonlinear

Model Order Reduction

Model reduction or model order reduction is a mathematical theory to find a low-dimensional approximation for a system of ordinary differential equations (ODEs). The main idea is that a high-dimensional state vector is actually belongs to a low-dimensional subspace as shown in Fig. 1.

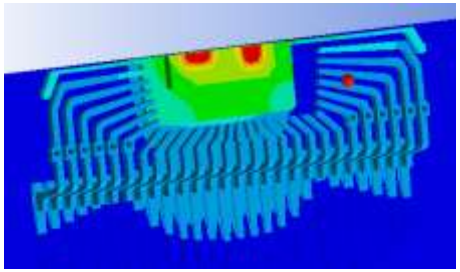
$$\mathbf{x} = V\mathbf{z} + \varepsilon$$

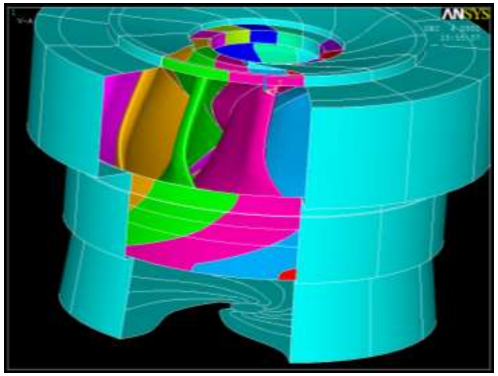
Fig. 1. Idea of a low-dimensional subspace.

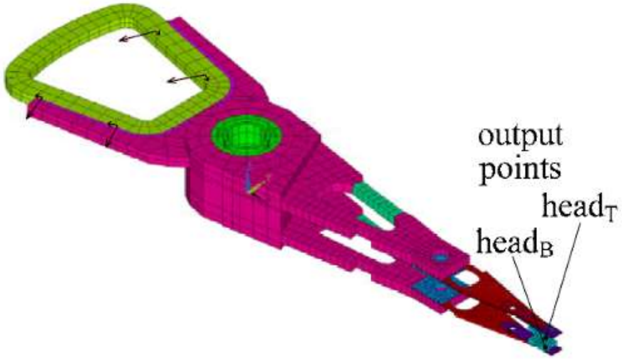
CADFEM

Content

- System level simulation and compact modeling
- Model order reduction
- Case studies with MOR for ANSYS
- Experience with MUMPS solver







CADFEM

MOR for ANSYS: http://ModelReduction.com

ANSYS Model

FULL files

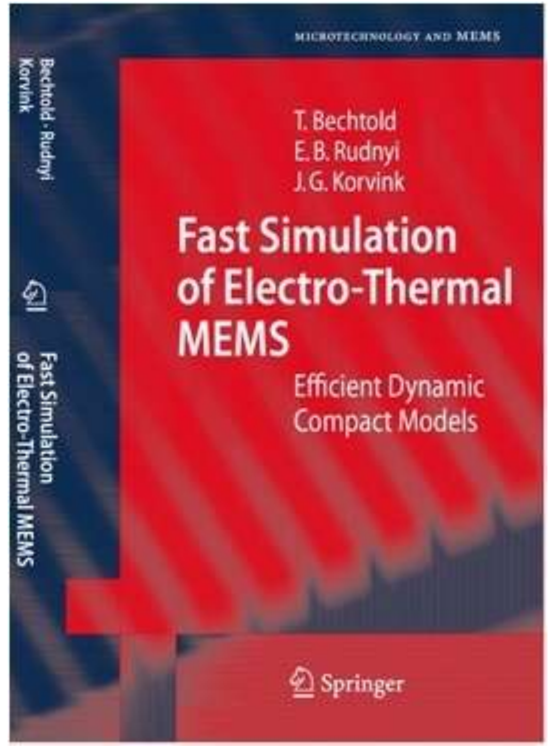
$$M\ddot{x} + E\dot{x} + Kx = Bu$$
$$y = Cx$$

Linear Dynamic System, ODEs

MOR Algorithm

Small dimensional matrices

Simulink, Simplorer, VerilogA, …



Current version 2.5

CADFEM

Implicit Moment Matching

- Take matrix and vector corresponding to a given expansion point.

$$s_0 = 0 \qquad P = A^{-1}E \qquad r = A^{-1}b$$

$$V = span\{\Im(A^{-1}E, A^{-1}b)\}$$

- Compute the orthogonal basis by the Arnoldi process.

$$V = span\{r, Pr, P^2 r, \ldots P^{k-1} r\}$$

- Project the original system on this basis.
- One can prove that this way the reduced model matches *k* moments.

- Multiple inputs:
  - Block Krylov Subspace;
  - Block Arnoldi;
  - Superposition Arnoldi.

Treating Matrix Inverse

- The Arnoldi process requires matrix vector product.
- Yet, we have a matrix inverse.

$$v_{i+1} = A^{-1} E v_i$$

- Instead solve a system of linear equations.

$$u_{i+1} = A^{-1} u_i$$

- This is the biggest computational cost:
  - Number of vectors x time for linear solve.

$$A u_{i+1} = u_i$$

$\Rightarrow$ The only right hand side is different.

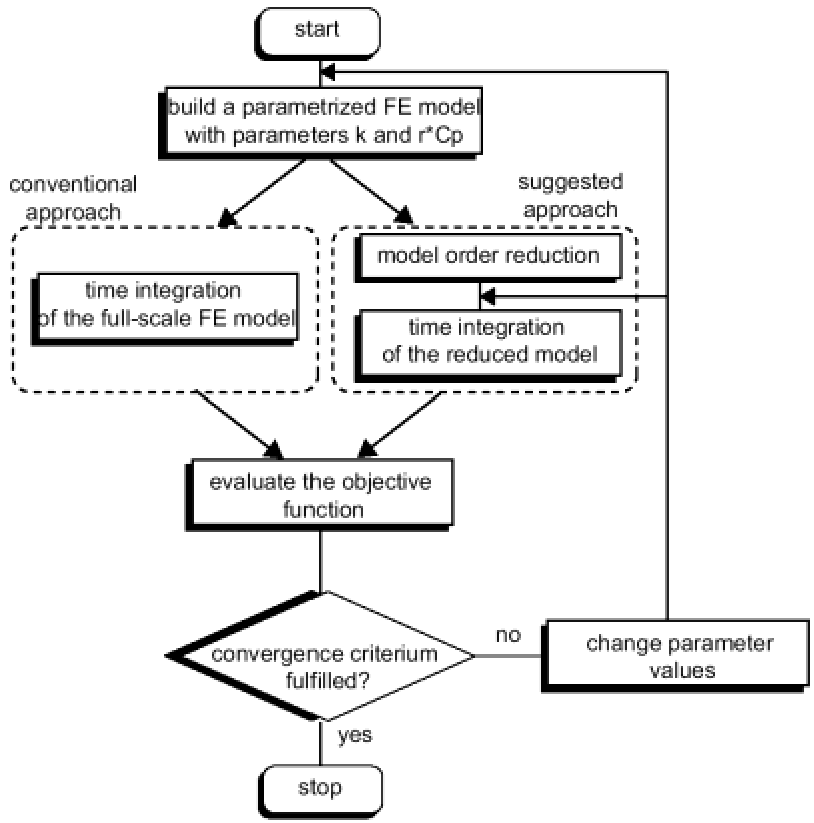
$\Rightarrow$ Can be used to speed it up.

- Direct solver has strong advantage.

MOR for ANSYS Timing: MOR as Fast Solver

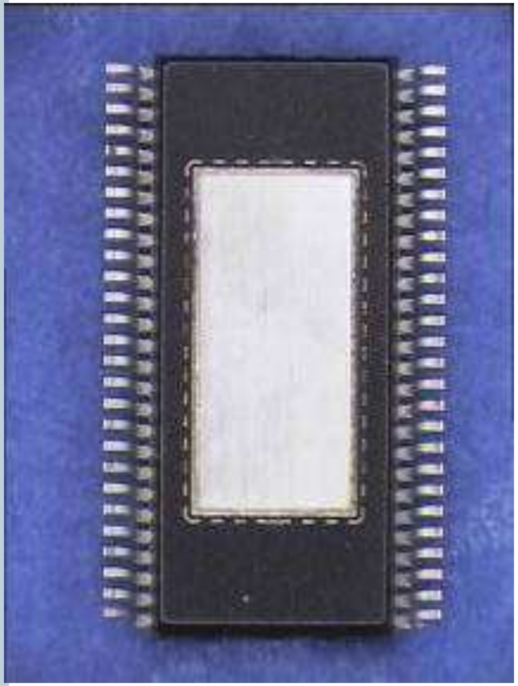
- Reduced model of dimension 30

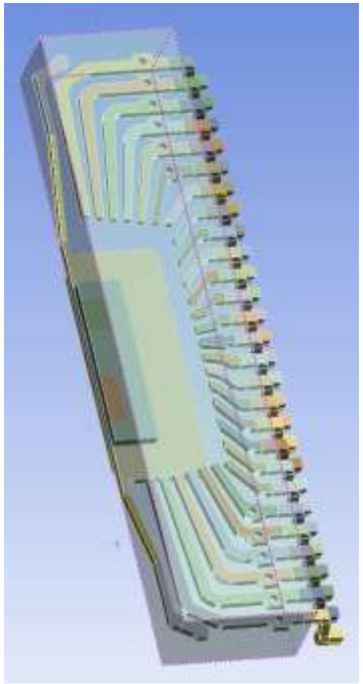
Dimension, DoFs	nnz	MOR Time /ANSYS static
4 267	20 861	1.4
11 445	93 781	1.8
20 360	265 113	1.7
79 171	2 215 638	1.5
152 943	5 887 290	2.2
180 597	7 004 750	1.9
375 801	15 039 875	1.7

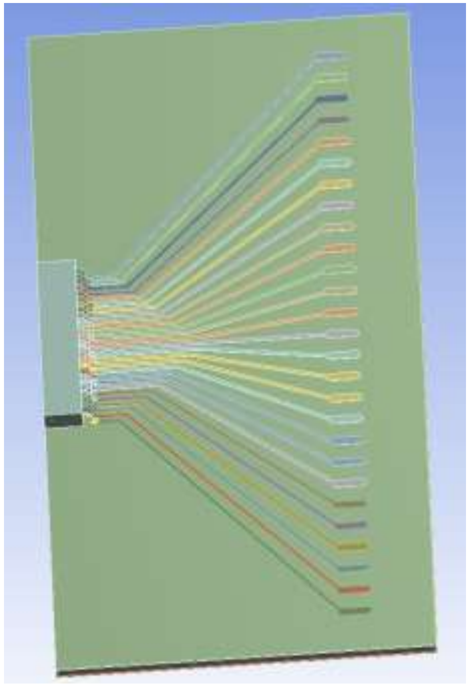


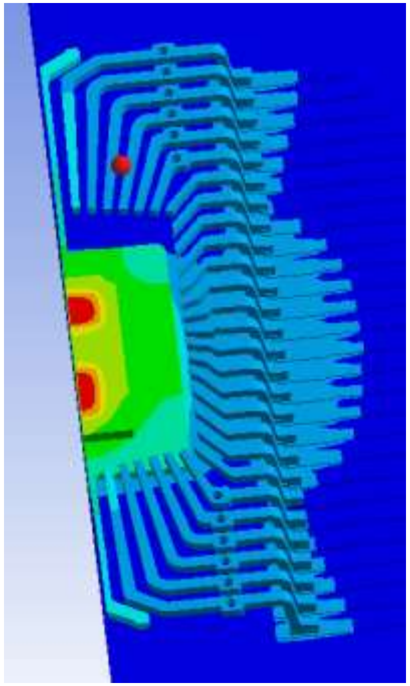
CADFEM

Chip and its Model in Workbench









freescale™ semiconductor

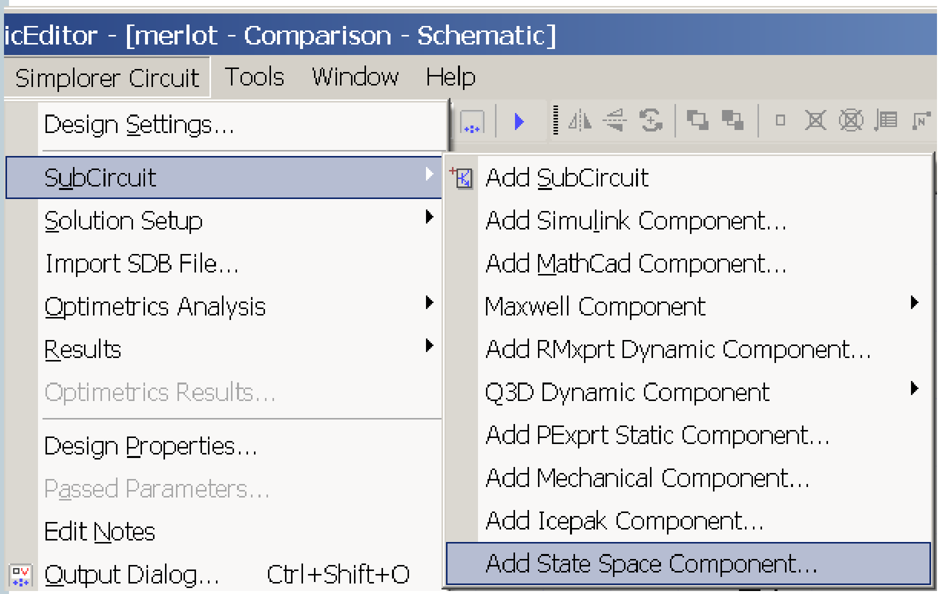
- Two power MOSFET transistors

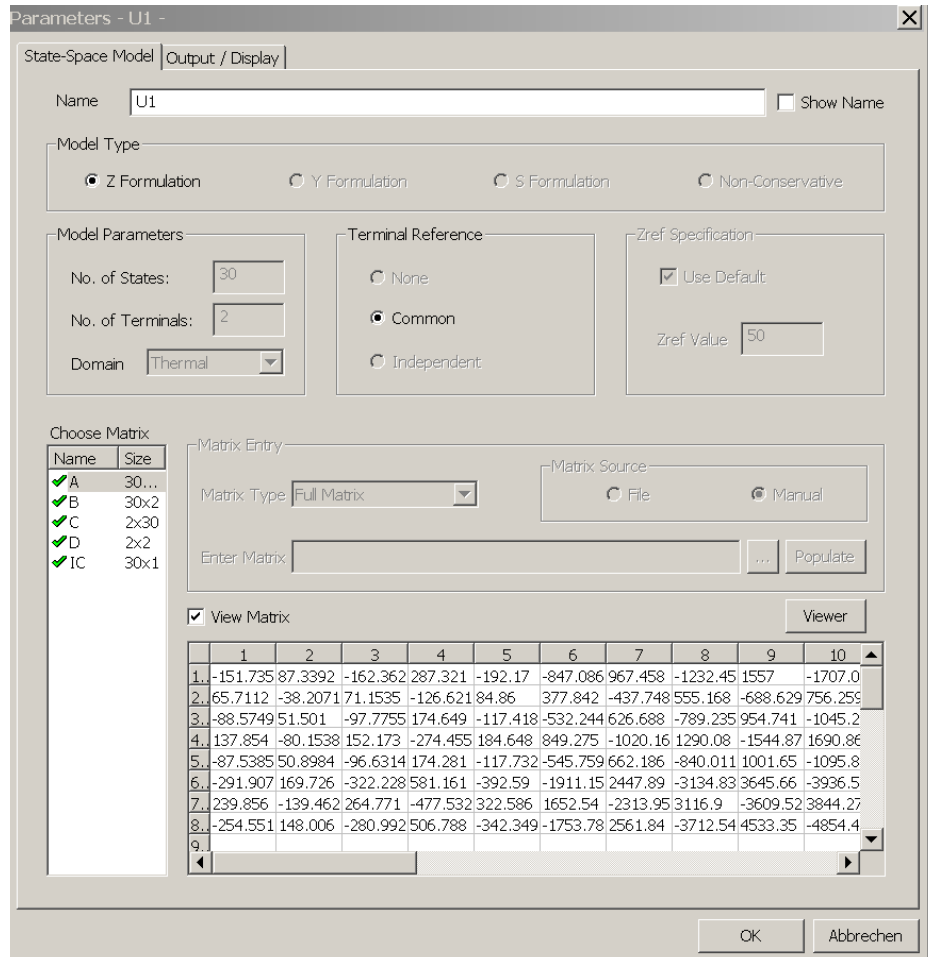
The example is from T. Hauck, I. Schmadlak, L. Voss, E. B. Rudnyi. *Electro-Thermal Simulation of Multi-channel Power Devices: From Workbench to Simplorer by means of Model Reduction*. NAFEMS, Seminar: Multi-Disciplinary Simulations - The Future of Virtual Product Development, 2009

CADFEM

Import Reduced Model in Simplorer

- Simplorer supports state space model

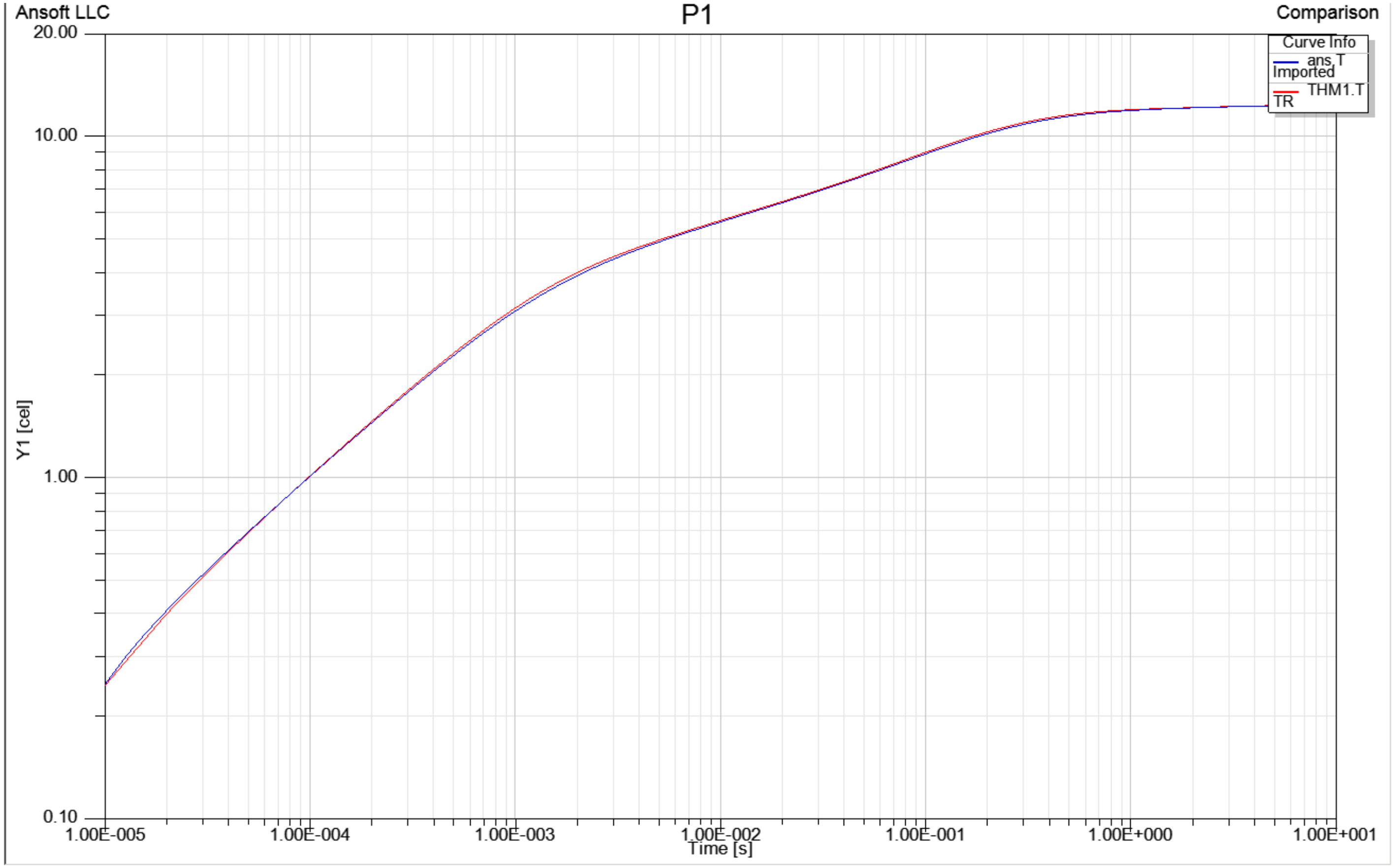




CADFEM

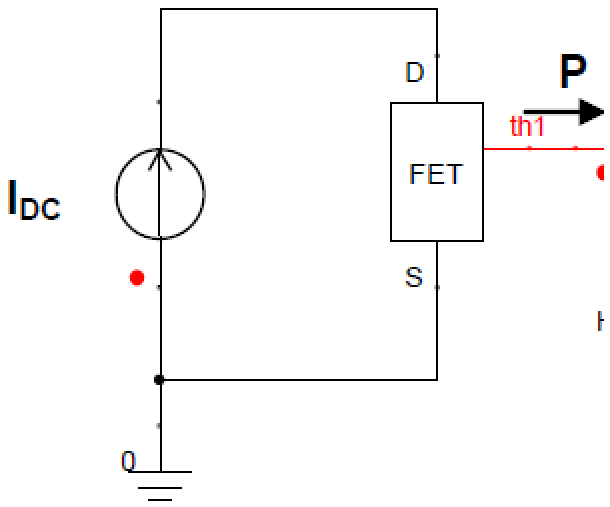
Thermal Impedance and Comparison with ANSYS

- ANSYS: about 300 000 DoFs, Reduced model: 30 DoFs
  - The difference is less than 1%
  - Timing: 60 timesteps is about 30 min in ANSYS



Thermal Runaway

- Transistor is considered as temperature dependent R_{DSon}
  - The VHDL-AMS model



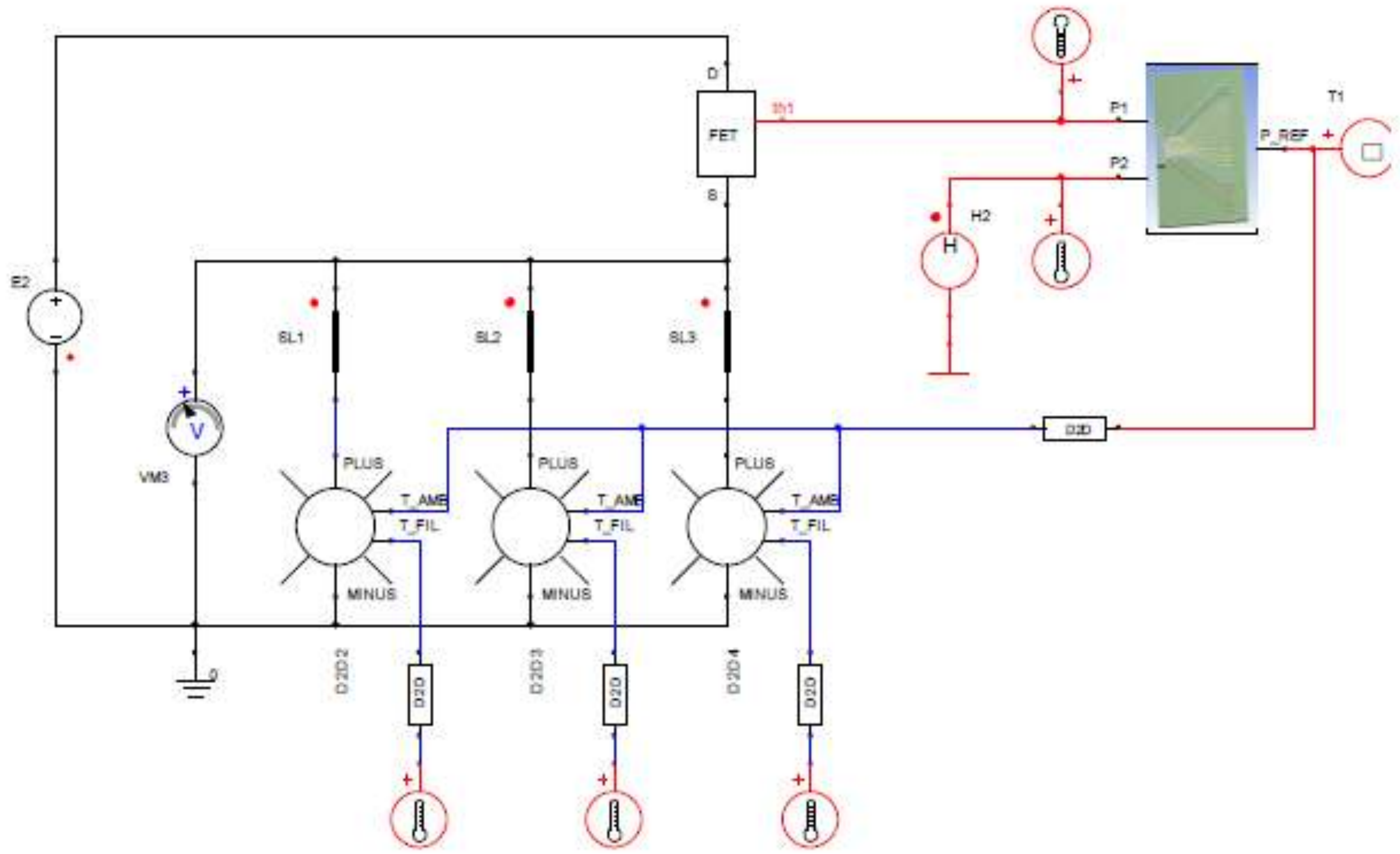

```
LIBRARY IEEE;
USE IEEE.ELECTRICAL_SYSTEMS.ALL;
USE IEEE.THERMAL_SYSTEMS.ALL;

ENTITY RDS_MODEL IS
   PORT (          QUANTITY RDS1 : RESISTANCE := 0.035;
                   QUANTITY t0 : IN TEMPERATURE := 298.0;
                   QUANTITY KC1 : REAL := 0.35e-3;
                   QUANTITY CTRL: REAL := 0.0;
                   TERMINAL th1 : thermal;
                   TERMINAL p,m  : ELECTRICAL);
END ENTITY RDS_MODEL;

ARCHITECTURE behav OF RDS_MODEL IS
                   QUANTITY v ACROSS i THROUGH p TO m;
                   QUANTITY t_val ACROSS h THROUGH th1 TO thermal_ref;
BEGIN
          IF (CTRL <= 0.0) USE
                   i == 0.0;
                   h == 0.0;
          ELSE
                   v == i*(RDS1+KC1*(t_val-t0));
                   h == -i*v;
          END USE;
END ARCHITECTURE behav;
```

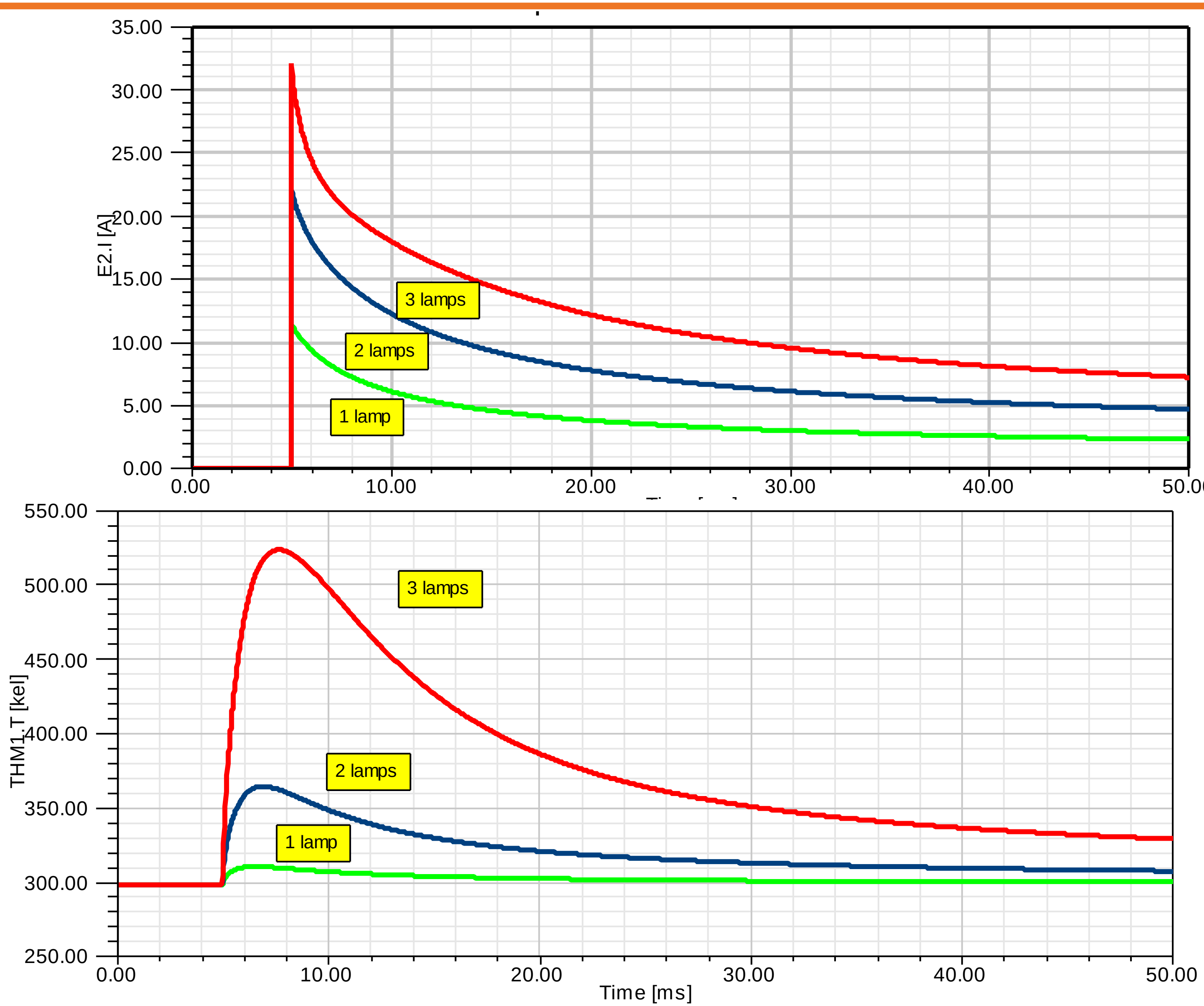

CADFEM

Transient Turn-on of an Automotive Light-Bulb



CADFEM

Transient Turn-on of an Automotive Light-Bulb



MOR for ANSYS in turbine dynamics

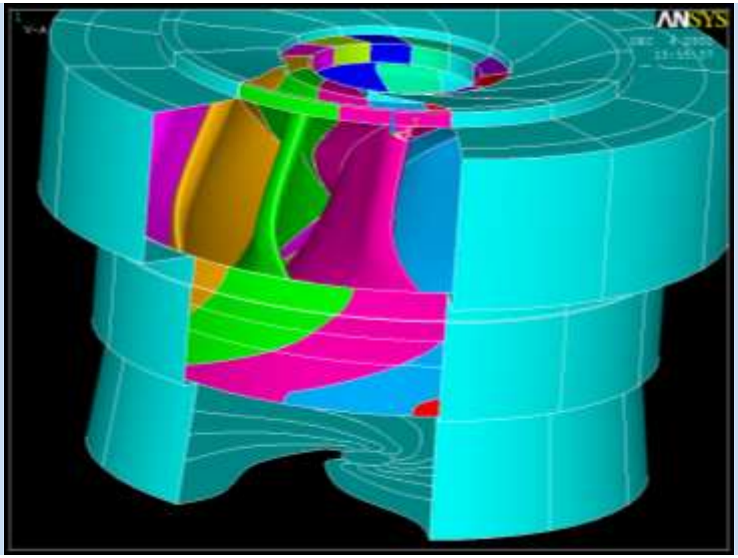
- Felix Lippold / Dr. Björn Hübner
- Voith Hydro Holding

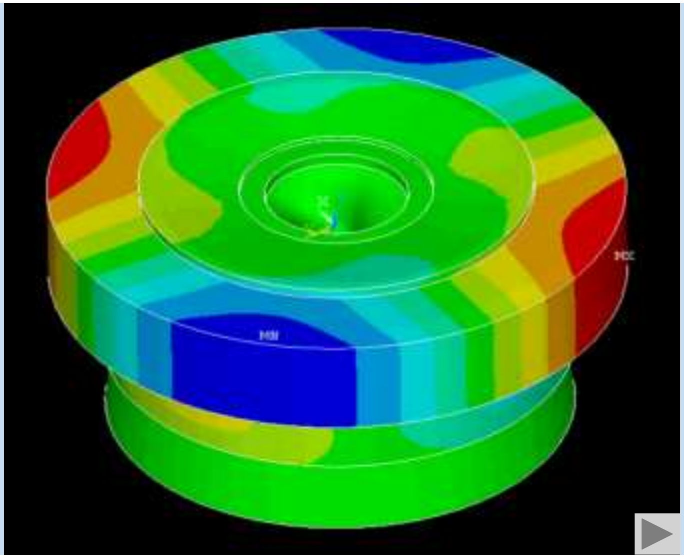
ANSYS Conference & 27. CADFEM Users Meeting, 18 - 20 November 2009, Congress Center Leipzig.

Modelling
Acoustic-structure coupling

Finite-Element model used in harmonic analysis

- Linear-elastic SOLID elements for runner structure.
- Acoustic FLUID elements for surrounding water.
- Complex load vectors for rotating pressure fields.





Modelling
Coupled acoustic-structure equations

Equations of motion

$$\begin{bmatrix} \mathbf{M}_\text{S} & \\ \mathbf{M}_\text{SA} & \mathbf{M}_\text{A} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{E}_\text{S} & \\ & \mathbf{E}_\text{A} \end{bmatrix} \cdot \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_\text{S} & \mathbf{K}_\text{SA} \\ & \mathbf{K}_\text{A} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_\text{S} \\ \mathbf{b}_\text{A} \end{bmatrix} \cdot q(t)$$

Formally equivalent to

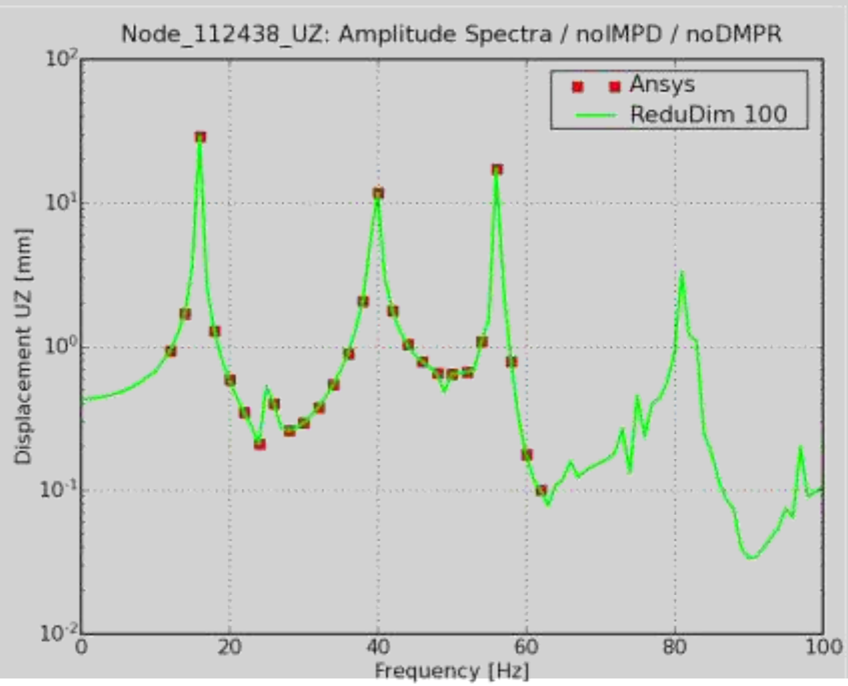
$$\mathbf{M} \cdot \ddot{\mathbf{x}} + \mathbf{E} \cdot \dot{\mathbf{x}} + \mathbf{K} \cdot \mathbf{x} = \mathbf{b} \cdot q(t)$$

But:

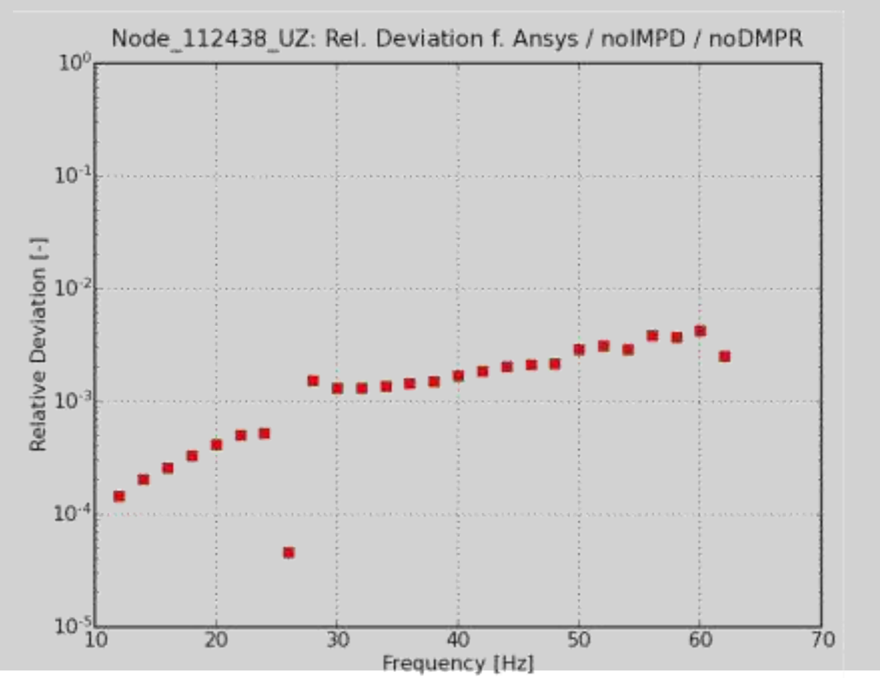
- non-symmetric matrices
- non-proportional damping matrix
- fluid damping $\mathbf{E}_\text{A}$ only for impedance boundaries

Results
No damping

Axial displacement@centre trailing edge – no damping



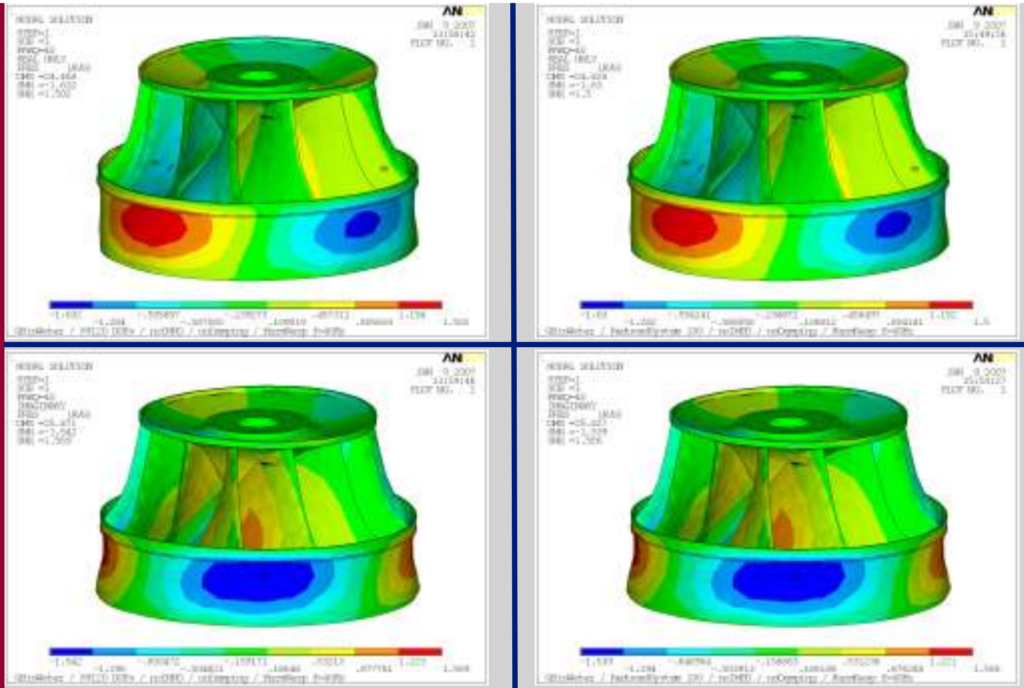
Amplitude spectrum (Dim=100)



Deviation related to ANSYS results

Results
Verification of reduced order method

Pressure distribution on runner for f = 40 Hz



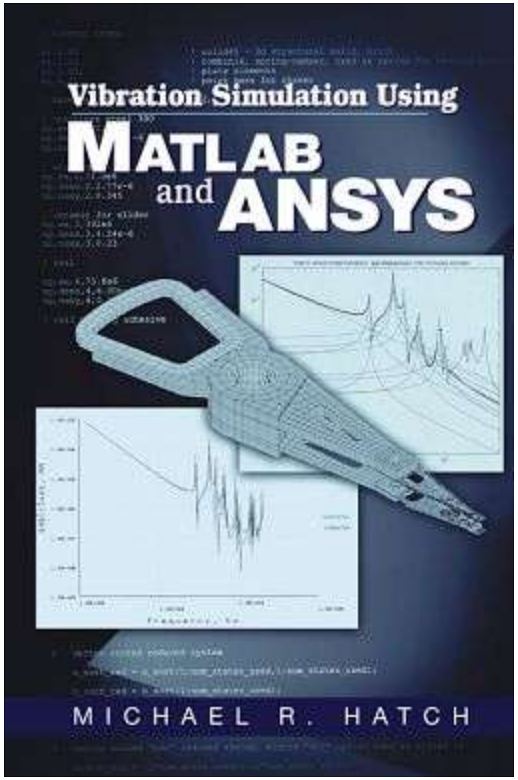
REAL part of pressure solution
Deviation < 0.2%

IMAG part of pressure solution
Deviation < 0.2%

Ansys (90 000 DOFs) | Reduced (Dim=100)

Hard Disk Drive Actuator/Suspension System

- Michael R Hatch, Vibration Simulation Using MATLAB and ANSYS



CADFEM

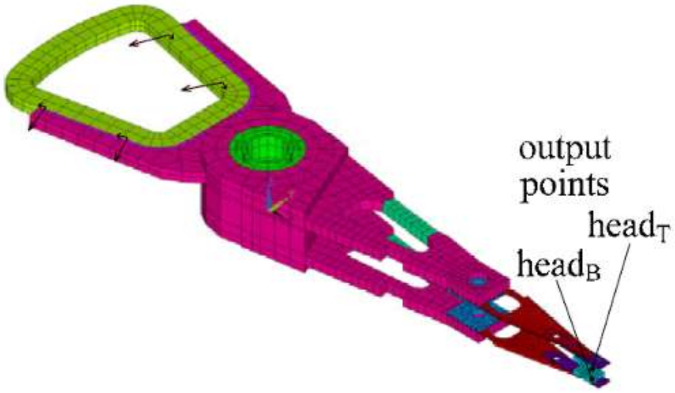
Model Reduction

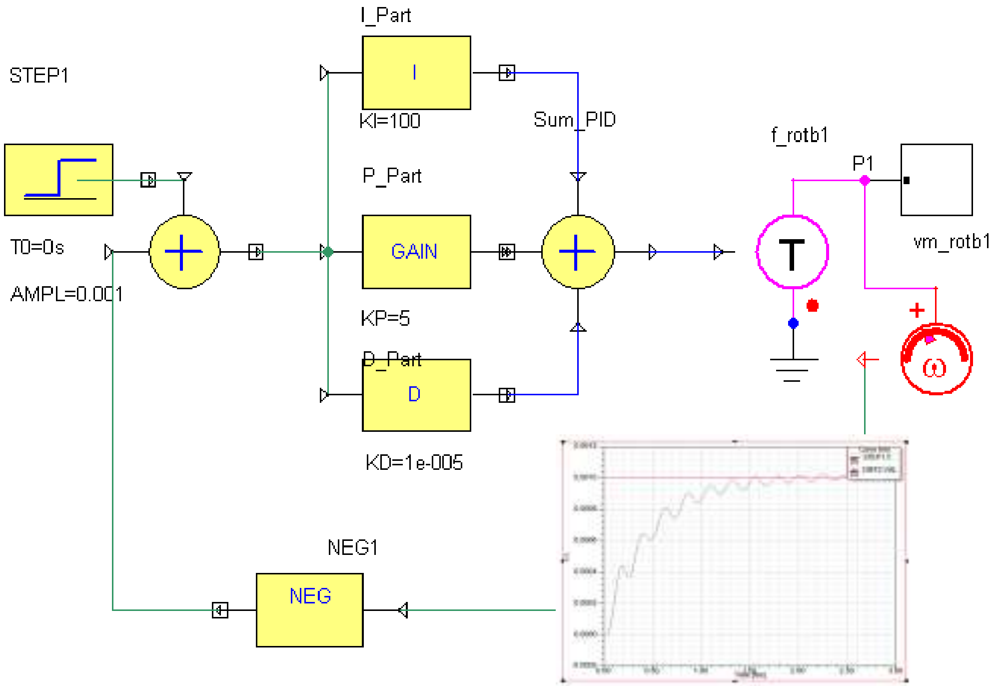
- 3352 elements
- 7344 nodes
- 21227 equation

- 400 frequencies takes about 12 min

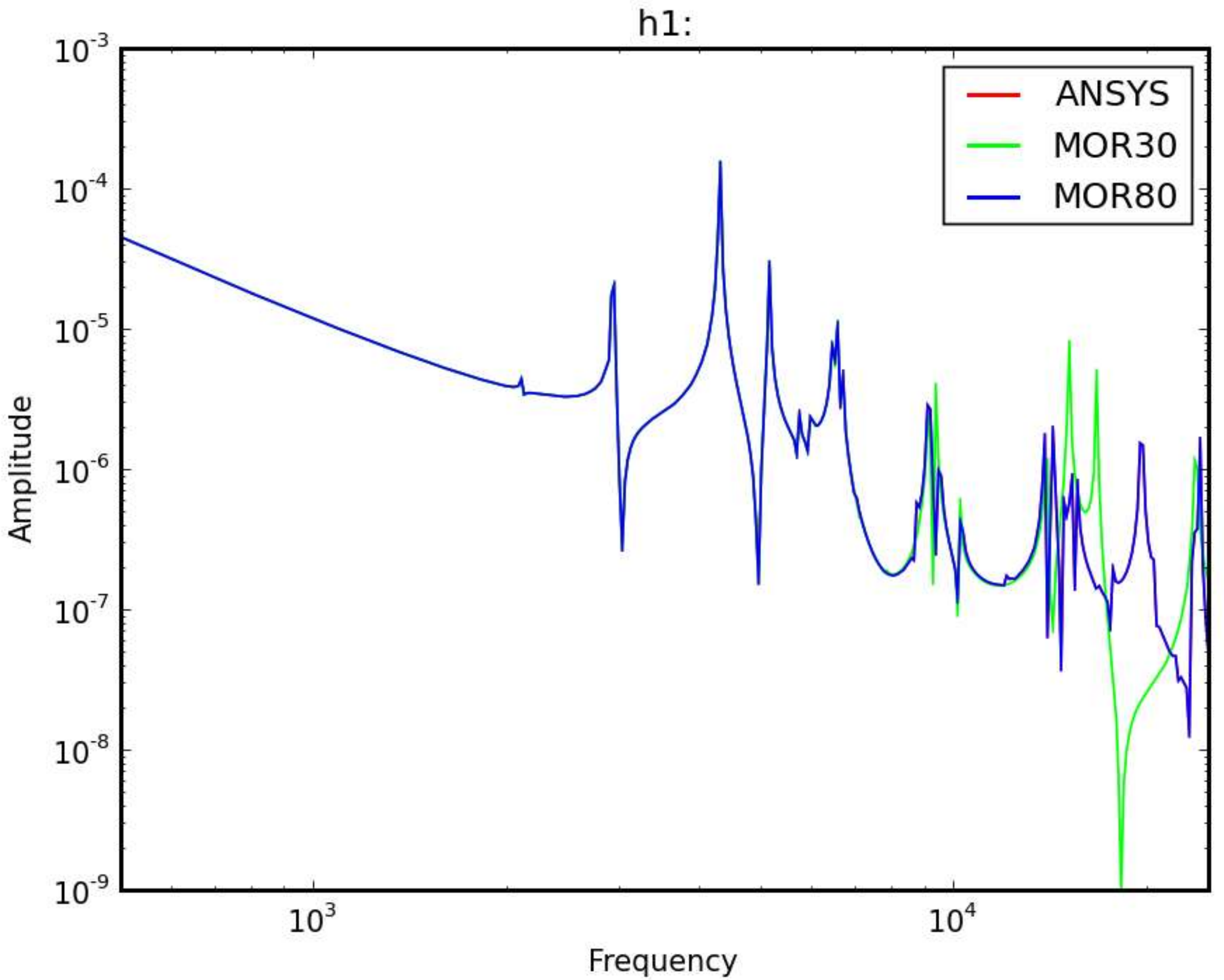
- MOR takes only 3 s

- Comparison for head
  - ANSYS
  - MOR 30
  - MOR 80

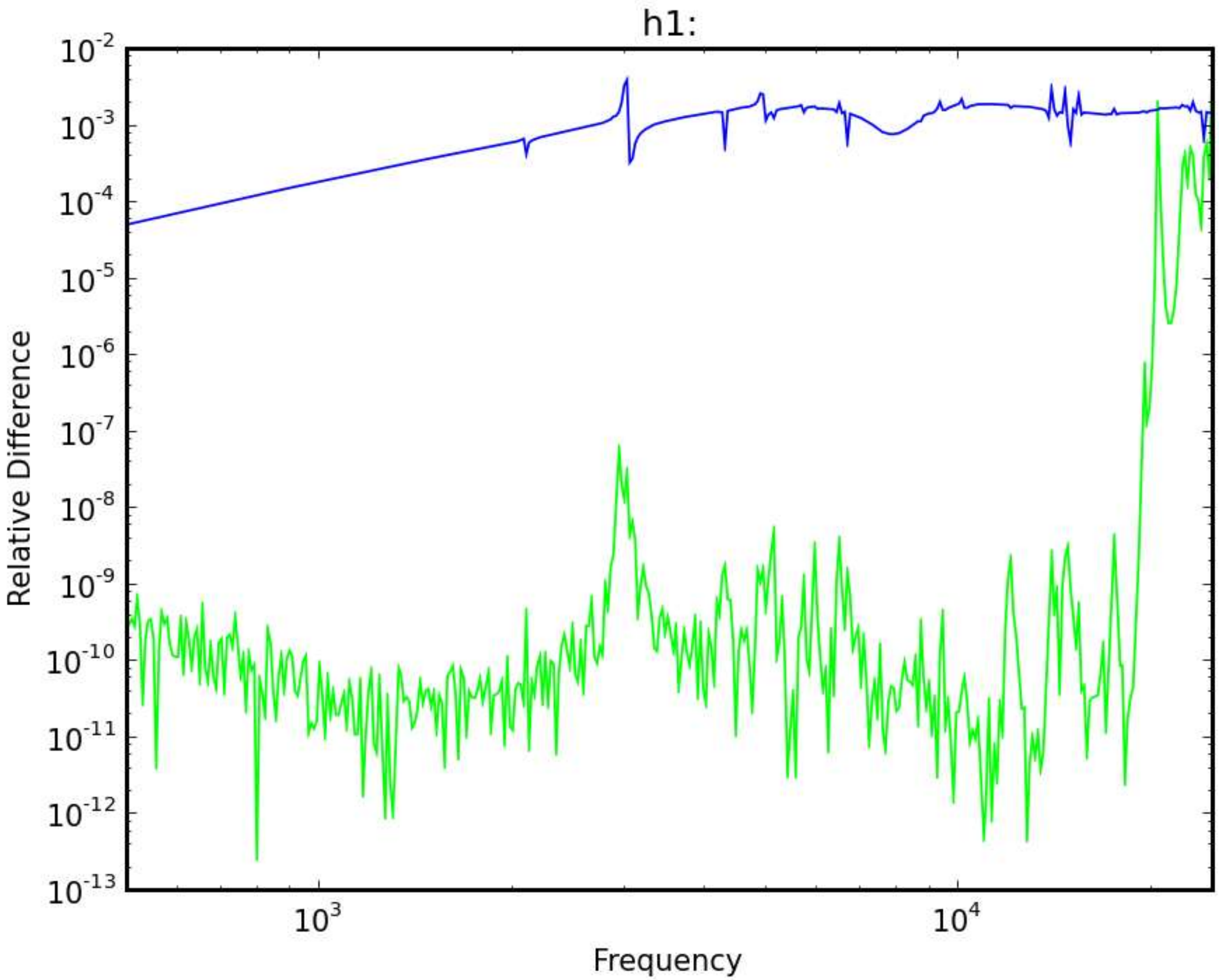




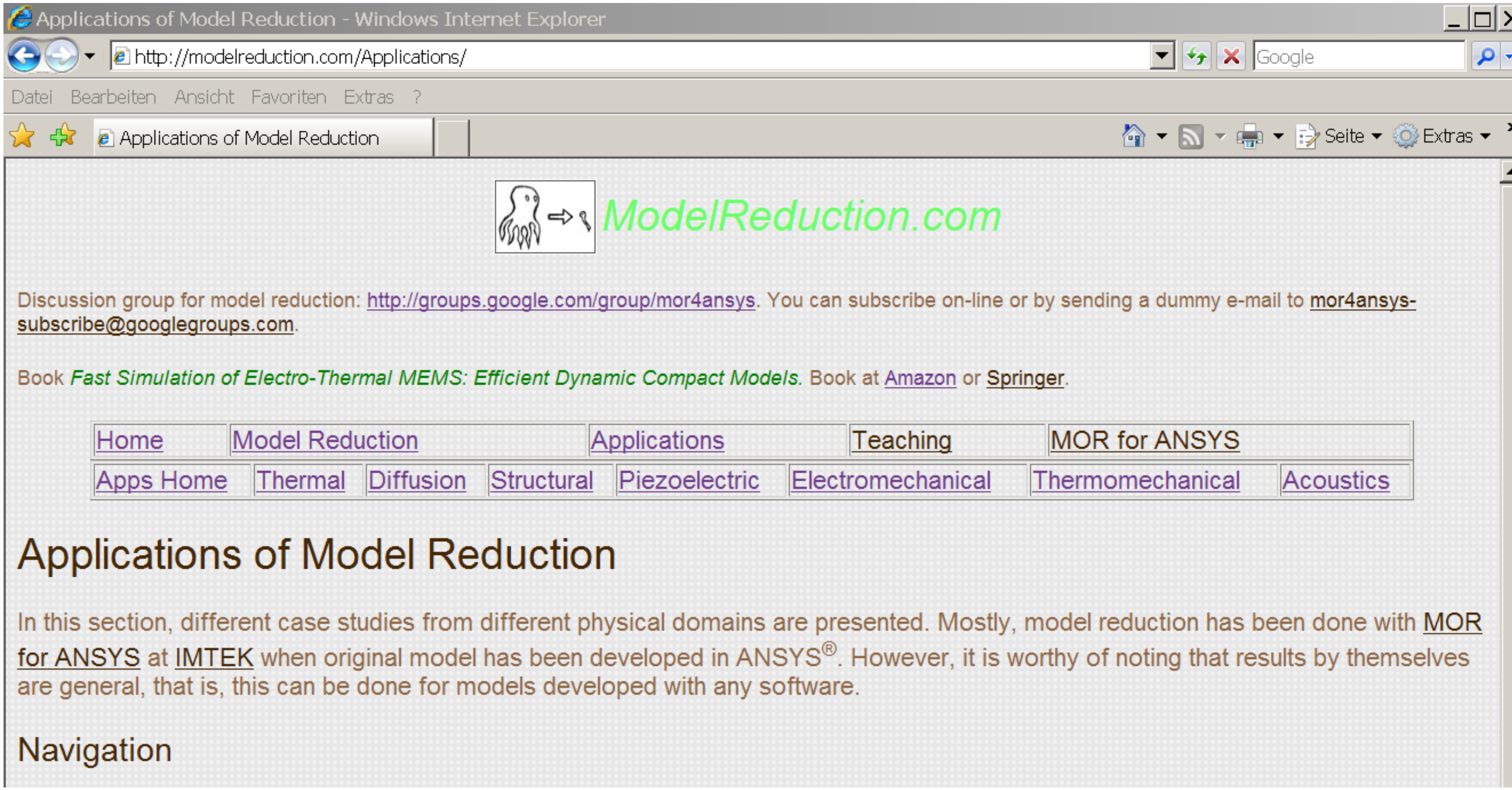
Comparison


h1:
ANSYS
MOR30
MOR80
Amplitude
Frequency
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{3}
10^{4}

Relative difference (blue modal80 – green MOR80)



Information on ModelReduction.com



Content

- CADFEM
- System level simulation and compact modeling
- Model order reduction
- Case studies with MOR for ANSYS
- Experience with MUMPS solver

www.msnbc.msn.com/id/12409082/

CADFEM

History of solvers in MOR for ANSYS

- MUMPS
  - Fortran 90 – Activation barrier

- TAUCS – symmetric matrices
- UMFPACK – unsymmetric matrices

- 2006 – HPC-EUROPA
  - SGI Origin 3800 (teras)
  - Micro-FEM bone models
    - Bert van Rietbergen

- 2007 – MUMPS in MOR for ANSYS

FREELY AVAILABLE SOFTWARE FOR - Windows Internet Explorer

http://www.netlib.org/utk/people/JackDongarra/la-sw.html

Datei Bearbeiten Ansicht Favoriten Extras ?

FREELY AVAILABLE SOFTWARE FOR

ScaLAPACK	BSD	yes	X	X	X	X			M/P
Trilinos/Pliris	LGPL	yes	X	X		X	X		M
SPARSE DIRECT SOLVERS	License	Support	Real	Complex	f77	c	c++	Seq	Dist
DSCPACK	?	yes	X			X		X	M
HSL	Own	yes	X	X	X			X	
MFACT	?	yes	X			X		X	M
MUMPS	PD	yes	X	X	X	X		X	M
PSPASES	?	yes	X		X	X			M
Quern	PD	yes	X			X	X	X	
SPARSE	?	?	X	X		X		X	
SPOOLES	PD	?	X	X		X		X	M
SuperLU	Own	yes	X	X	X	X		X	M
TAUCS	Own	yes	X	X		X		X	
Trilinos/Amesos	LGPL	yes	X					X	M
UMFPACK	LGPL	yes	X	X		X		X	
Y12M	?	yes	X		X			X	
PRECONDITIONERS	License	Support	Real	Complex	f77	c	c++	Seq	Dist
BPKIT	?	yes	X		X	X	X	X	M
MLD2P4	BSD	yes	X	X	f90			X	M
MSPAI	LGPL	yes	X	X			X	X	M
PARPRE	?	yes	X			X			M
SPAI	?	yes	X			X		X	M
Trilions/ML	LGPL	yes	X	X		X	X	X	M

CADFEM

Voith Hydro Timing

- 2 x quad-core Intel X5560 (Nehalem) with 48 GB under Windows
  - Shared memory

	nnz	dim
K	91627731	1292663
M	31982656	1292663

- 27.7 Gb memory
- Factorization is 287 s
- 250 vectors is 1500 s
  - 6 s pro vector

	nnz	dim
K	99934728	1495341
M	35774706	1495341

- 20.1 Gb memory
- Factorization is 243 s
- 200 vectors is 1300 s
  - 6.5 s pro vector

How MUMPS is used in MOR for ANSYS

- Shared memory (serial)
  - No parallel
- Linux and Windows versions (Visual C + Intel Fortran on Windows)
  - Used to have an Altix version
  - Intel MKL BLAS
    - Used to work with ATLAS
- Two versions:
  - 32-bit integers
  - 64-bit integers
- Mostly in-core, sometimes out-of-core
- Symmetric positive definite, symmetric indefinite, unsymmetric matrices

Sharing Experience

- Oberwolfach Model Reduction Benchmark Collection
  - http://portal.uni-freiburg.de/imteksimulation/downloads/benchmark
  - Trabecular Bone Micro-Finite Element Models

- http://MatrixProgramming.com
  - Compiling MUMPS under Microsoft Visual Studio and Intel Fortran with GNU Make
  - Sample code in C++ to run MUMPS

- Working with Microsoft Visual Studio
  - Distribution the code with manifests
  - -02 does not turn safe iterators off (_SECURE_SCL=0)
  - 32-bit code has some problems with memory fragmentation
    - Using a tree to assemble a matrix

Conclusion

- Model Order Reduction is a nice extension for a FEM application
- Model Order Reduction needs a direct solver
- MUMPS plays an important role in MOR