NULL SPACE COMPUTATION OF SPARSE SINGULAR MATRICES WITH MUMPS

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Abstract.
Computing a basis for the null space of a large and sparse matrix is often a key step of some algorithms in structural analysis, fluid mechanics, constrained optimization, image reconstruction or electrical engineering. Various algorithms based on rank revealing LU or rank revealing orthogonal decompositions have been proposed in the literature; see e.g. [1, 2, 3, 4, 5]. Nevertheless to the best of our knowledge the case of large sparse matrices in the framework of multifrontal methods has been rarely addressed.

In this talk we propose to use a sparse Gaussian Elimination of $A \in \mathbb{C}^{n\times n}$ to derive null space information by inspecting only $U \in \mathbb{C}^{n\times n}$, the singular upper triangular factor obtained after numerical factorization of the preprocessed matrix $\tilde{A} = P_s A P_c P_s^T$, where P_s corresponds to a permutation that aims at minimizing the fill-in during factorization and P_c is a column permutation to obtain a zero-free diagonal. Thus we would like to determine accurately the deficiency of U and a basis of its null space. We will detail the two main modifications included in a multifrontal method to derive a (hopefully) rank-revealing sparse Gaussian Elimination algorithm. Finally we will analyse the behaviour of the proposed algorithm - implemented in MUMPS - on some singular matrices related to both academic problems and real-life applications.

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